Int. J. Heat Mass Transfer. Vol. 1, pp. 219–235. Pergamon Press 1960. Printed in Great Britain.

ON THE CORRELATION OF NUCLEATE BOILING HEAT TRANSFER

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(*Received* 22 *September* 1959; *revised* 8 *December* 1959)

Abstract—A generalized equation to describe nucleate boiling is derived. The expression which correlates all fluid independently of pressure and heating surface-fluid combination is as follows:

$$Y = 8{\cdot}0\,(f_\zeta^{1/2} f_p X)^{2/3}$$

where

$$X = \left(\frac{1}{M^2 P}\,\frac{c_p \gamma'^2}{\lambda \sigma r \gamma''}\right)^{1/2} R^{3/2} q$$

$$Y = \frac{\alpha R}{\lambda}$$

and

c_p = specific heat of liquid;
γ' and γ'' = specific weight of liquid and vapour;
λ = thermal conductivity of liquid;
σ = surface tension of liquid;
R = the representative length of heating surface;
q = heat flux;
α = coefficient of heat transfer;
$M = 900\ \mathrm{m}^{-1}$, $P = 1{\cdot}699$ kcal/hr.
f_ζ = foamability, and f_p = pressure factor.

Furthermore, it is indicated that this correlating equation is applicable to forced-convection saturated or surface boiling. The proposed correlating equation has been theoretically sought by analysing the elementary processes of phenomena, but it is not based on dimensional analysis. In order to reach the final solution of the problem, it is necessary to analyse the elementary processes of boiling phenomena and elucidate the relations underlying the problem.

Résumé—Une équation générale représentant le phénomène d'ébullition a été obtenue. Cette expression, valable pour tout fluide indépendamment de la pression et de la disposition de la surface chauffante est la suivante:

$$Y = 8{,}0(f_\zeta^{1/2} f_p X)^{2/3}$$

ou

$$X = \left(\frac{1}{M^2 P}\,\frac{c_p \gamma'^2}{\lambda \sigma r \gamma''}\right)^{1/2} R^{3/2} q$$

$$Y = \frac{\alpha R}{\lambda}$$

et

c_p = chaleur spécifique du liquide,
γ' et γ'' = poids spécifique du liquide et de la vapeur,
λ = conductibilité thermique du liquide,
σ = tension superficielle du liquide,

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R = longueur caractéristique de la surface chauffante,
q = flux de chaleur,
α = coefficient de transmission de chaleur,
M = $900\ \mathrm{m}^{-1}$, $P = 1{,}699$ kcal/h
f_ζ = pouvoir moussant, et f_p = facteur de pression.

De plus, il est indiqué que cette équation de corrélation est applicable à l'ébullition saturée ou de surface avec convection forcée. L'équation proposée a été obtenue théoriquement en analysant les processus élémentaires du phénomène, mais elle n'est pas fondée sur l'analyse dimensionnelle. Pour atteindre la solution finale du problème, il est nécessaire d'analyser les processus élémentaires du phénomène d'ébullition et d'expliquer les relations en approfondissant le problème.

Zusammenfassung—Zur Beschreibung des Filmsiedens wird eine verallgemeinerte Gleichung abgeleitet. Dieser Ausdruck, der für alle Flüssigkeiten, aber unabhängig vom Druck und unabhängig von der Kombination Heizfläche-Flüssigkeit, gilt, lautet:

$$Y = 8{,}0(f_\zeta^{1/2} f_p X)^{2/3}$$

Darin ist

$$X = \left(\frac{1}{M^2 P}\,\frac{c_p \gamma'^2}{\lambda \sigma r \gamma''}\right)^{1/2} R^{3/2}\, q$$

$$Y = \frac{\alpha R}{\lambda}$$

und

c_p = spezifische Wärmekapazität der Flüssigkeit
γ' und γ'' = Gewicht von Flüssigkeit und Dampf
λ = Wärmeleitfähigkeit der Flüssigkeit
σ = Oberflächenspannung der Flüssigkeit
R = Kennzeichnende Länge der Heizfläche
q = Wärmestromdichte
α = Wärmeübergangskoeffizient
M = $900\ \mathrm{m}^{-1}$, $P = 1{,}699$ kcal/h
f_ζ = Schaumfähigkeit, und f_p = Druckfaktor

Diese Beziehung ist auch auf die erzwungene Konvektion in gesättigter oder unterkühlter Flüssigkeit (örtliches Sieden) anwendbar. Sie wurde durch eine theoretische Untersuchung der Elementarvorgänge gefunden und beruht nicht auf einer Dimensionsanalyse. Um die endgültige Lösung des Problems zu erhalten, muss man die Beziehungen aufklären, die den Elementarprozessen des Siedevorgangs zu Grunde liegen.

Abstract—Выведено обобщенное уравнение, описывающее пузырчатое кипение. Выражение, устанавливающее корреляционную связь для всей жидкости независимо от давления и комбинации «нагревающая поверхность–жидкость», имеет следующий вид:

$$Y = 8{,}0\,(f_\zeta^{1/2} f_p X)^{2/3}$$

где

$$X = \left(\frac{1}{M^2 P}\,\frac{c_p \gamma'^2}{\lambda \sigma r \gamma''}\right)^{1/2} R^{3/2}\, q$$

$$Y = \frac{\alpha R}{\lambda}$$

и

c_p = удельная теплоёмкость жидкости,
γ' и γ'' = удельный вес жидкости и пара соответственно,
λ = теплопроводность жидкости,
σ = поверхностное натяжение жидкости,
R = определяющая длина нагревающей поверхности,
q = тепловой поток,
α = коэффициент теплопереноса,

M = 900 м$^{-1}$, P = 1,699 ккал/час,
f_ζ = способность к образованию пузырей, и f_p = коэффициент давления.

Кроме того, указывается, что корреляционное уравнение может быть применено к насыщенному кипению при вынужденной конвекции или плёночному кипению. Предлагаемое корреляционное уравнение было выведено теоретически путём анализа элементарных процессов рассматриваемых явленьий, а не путём анализа размерностей. Для того чтобы получить окончательное решение задачи, необходимо проанализировать элементарные процессы, из которых складывается кипение, и объяснить лежащие в основе задачи соотношения.

NOMENCLATURE

A,	area of heating surface (m^2);
D,	diameter of tube (m);
H,	liquid level (m);
H_e,	effective stirring length of bubbles in saturated boiling (m);
H_{es},	effective stirring length of bubbles in surface boiling (m);
R,	representative dimension of heating surface (m);
U,	rising velocity of a bubble at the optional point (m/hr);
U_m,	average rising velocity of a bubble (m/hr);
V,	volume of a rising bubble at the optional point (m^3);
V_0,	volume of a bubble just leaving the heating surface (m^3);
W,	total convective driving force for heat transfer (m);
W_t,	average convective driving force due to the change of density (m);
W_b,	average convective driving force due to the stirring of bubbles (m);
c_p,	specific heat of liquid (kcal/kg°C);
d_0,	diameter of a bubble just leaving the heating surface (m);
d_u,	diameter of a bubble just leaving the free liquid surface (m);
f,	frequency of bubble formation (sec^{-1});
f_p,	pressure factor;
f_ζ,	foamability;
g,	acceleration due to gravity (m/sec^2);
h,	rising height of a bubble (m);
n,	number of vapour columns;
p,	any pressure (kg/cm^2);
p_s,	atmospheric pressure (kg/cm^2);
q,	heat flux of heating surface (kcal/m^2 hr);
q_c,	convective heat flux (kcal/m^2 hr);
q_b,	saturated boiling heat flux (kcal/m^2 hr);
r,	latent heat of evaporation (kcal/kg);
v,	velocity of fluid (m/sec);
α,	coefficient of heat transfer (kcal/m^2 hr °C);
α_c,	coefficient of convective heat transfer (kcal/m^2 hr °C);
α_b,	coefficient of boiling heat transfer (kcal/m^2 hr °C);
β,	coefficient of cubic expansion (°C^{-1});
γ',	specific weight of liquid (kg/m^3);
γ'',	specific weight of vapour (kg/m^3);
δ,	thickness of temperature boundary layer (mm);
ζ,	coefficient of foaming capacity of the combination of the surface and liquid in question (kcal/m hr °C^3);
ζ_s,	coefficient of foaming capacity of pure liquid on the fresh and smooth surface (kcal/m hr °C^3);
η,	viscosity of liquid (kg hr/m^2);
θ_L,	temperature of liquid (°C);
θ_w,	temperature of heating surface (°C);
θ_s,	saturation temperature of liquid (°C);
$\Delta\theta$,	temperature difference between heating surface and liquid (°C);
$\Delta\theta_s$,	surface temperature minus saturation temperature (°C);
λ,	thermal conductivity of liquid (kcal/m hr °C);
ν,	kinematic viscosity of liquid (m^2/hr);
σ,	surface tension of liquid (kg/m);
τ,	period of bubble formation (sec);
χ,	(diameter of a bubble just leaving the heating surface) (frequency of bubble formation) (m/hr);
Gr,	Grashof number;
Gr^*,	equivalent Grashof number;
Nu,	Nusselt number;
Pr,	Prandtl number;
Re,	Reynolds number;
$\bar{R}e_b$,	bubble Reynolds number.

1. INTRODUCTION

THE study of heat transfer in a boiling liquid is one of the most neglected in the field of heat engineering. An effort to unify heat transfer in nucleate boiling of various kinds of liquids that have different physical properties into one correlating equation is being made by a few research workers, but the equation concerned with the irreproducibility of phenomenon has yet to be perfected. The difficulties in the investigation of boiling can be attributed to the fact that boiling is not only a complex process involving fluid motion, heat transfer, surface phenomenon and phase change, but that it is also irreproducible.

Even if a liquid is always boiled in one and the same vessel under the same conditions of heating, sometimes a few bubbles are generated and rise expanding rapidly and, at other times, a great number of small bubbles rise quietly, thus causing much difference in the phases of heat transfer in each individual case. Before studying the origin of this irreproducibility of boiling, it is necessary first to analyse the composite phenomena of boiling into elementary processes. It is only after this analysis that the problem of irreproducibility as well as the nature of boiling phenomenon will be clarified.

It is a fact of course that the problem of heat transfer holds the key to the consideration of boiling phenomena from the standpoint of engineering. However, to rely on that viewpoint is to develop as a whole the relations between the coefficient of heat transfer and various other quantities by means of dimensional analysis rather than by treating the various elementary processes analytically. For instance, McAdams [15] collected the existing data in his book. This synthetic treatment seems to be a direct method, but the problem of irreproducibility cannot be solved extensively by it. Moreover, in the application of dimensional analysis, there will arise questions of what dimensionless quantities to select. It is because of this ambiguity that empirical equations hitherto proposed are different from one another in the introduction of dimensionless quantities.

In order to eliminate such disadvantages, a law which is valid for individual elementary process should be confirmed and then unified. The reason why Jakob's studies [11] are held in high esteem even now is that his study was the only one that took a correct course, though it still leaves much to be clarified. Rising current interest in research into the bubble dynamics of boiling may be due to the fact that the pursuit of the elementary processes involving generation, growth and collapse of bubbles is the key to the clarification of boiling phenomena. As there is a limit to the complete theoretical explanation of every elementary process, it is necessary to study each one experimentally.

In this paper we consider the derivation of the correlating equation of nucleate boiling heat transfer based on the above-mentioned viewpoint and its application to the forced-convection saturated or surface boiling.

2. DERIVATION OF CORRELATING EQUATION

Consider the case of pool boiling of a saturated liquid. Two kinds of driving force for convection at nucleate boiling, are conceivable. The first driving force of convection is the buoyancy due to the change of density accompanying heating, similar to the case of purely free convection. The buoyancy that acts upon the unit weight is plainly expressed as follows:

$$\beta\,(\theta - \theta_L)$$

and average convective driving force W_t against the whole liquid will be expressed as follows in the form:

$$W_t = \int_0^\delta \beta(\theta - \theta_L)\,dy \qquad (1)$$

where

$\beta =$ coefficient of cubic expansion of liquid;

$y =$ vertical distance from heating surface;

$\delta =$ average thickness of the boundary layer of the liquid along the heating surface;

$\theta =$ temperature of the liquid at the point $y(<\delta)$;

$\theta_L =$ temperature of the liquid at the point $y(>\delta)$.

The second driving force of convection is the liquid stirring force of rising bubbles and can be considered as follows. Let:

$V =$ volume of one rising bubble at the optional point y;

U = rising velocity of the bubble at the optional point y;
τ = period of bubble formation;
A = heating surface area;
n = number of vapour columns.

The average volume of the vapour–liquid mixture containing one bubble at an optional height is $(A/n)U\tau$. Therefore, the convective force due to the change in bulk density acting on a unit weight of liquid may be expressed as follows, assuming that the weight of vapour be neglected compared with that of liquid:

$$nV/AU\tau$$

Accordingly the convective driving force W_b due to the stirring of bubbles for the entire liquid may be expressed as follows:

$$W_b = \int_0^{H_e} \frac{n}{A\tau} \frac{V}{U} F(y)\, dy \tag{2}$$

Here H_e is the "effective stirring length of bubbles". The effective stirring length of bubbles is the distance from the heating surface where the liquid stirring effect of bubbles disappears, for it is considered that when the bubbles rise from the heating surface, the nearer they are to the heating surface, the greater their stirring effect upon the boundary layer near the heating surface gets, and the greater the distance of bubbles from the heating surface is, the weaker their effect becomes. $F(y)$ represents the discrepancies of the liquid stirring effect due to the rising position of bubbles and its limiting value should be as follows:

$$F(0) = 1,\ F(H_e) = 0 \tag{3}$$

Since these two convective driving forces operate at the same time, the total convective driving force W may be expressed as follows:

$$W = W_t + W_b \tag{4}$$

A coefficient of no dimension will be defined as follows in order to express equations (1) and (2) in a simple way:

$$c_t = \int_0^1 \frac{\theta - \theta_L}{\theta_p - \theta_L}\, d\eta \quad , \quad \eta = \frac{y}{\delta} \tag{5}$$

$$c_b = \frac{1}{6} \int_0^{H_e} \frac{V(y)}{V_0} \frac{U_m}{U(y)} F(y) \frac{dy}{H_e} \tag{6}$$

If the above coefficients are used, equation (4) will be as follows:

$$W = c_t \beta(\theta_p - \theta_L)\, \delta + c_b \frac{nd_0^2}{R^2} \frac{\chi}{U_m} H_e \tag{7}$$

where
θ_p = average temperature of heating surface;
V_0 = volume of a bubble just leaving the heating surface;
U_m = average rising velocity of a bubble;
d_0 = diameter of a bubble just leaving the heating surface (transformed to a sphere);
R = representative dimension of heating surface (radius in case of horizontal circular plate);
χ = d_0/τ.

Since the following Grashof number

$$Gr = \frac{R^3 g\beta(\theta_p - \theta_L)}{\nu^2} = \frac{R^3 g}{\nu^2} \frac{W_t}{c_t \delta} \tag{8}$$

is used to express the driving force of a free convection W_t, the equivalent Grashof number may be introduced in this case, too:

$$Gr^* = \frac{R^3 g}{\nu^2} \frac{W}{c_t \delta} =$$

$$= Gr + \left(\frac{c_b}{c_t}\right) \left(\frac{nd_0^2}{R^2}\right) \left(\frac{\chi}{U_m}\right) \left(\frac{H_e}{\delta}\right) \left(\frac{gR^3}{\nu^2}\right) \tag{9}$$

where
Gr = Grashof number;
g = acceleration due to gravity;
ν = kinematic viscosity of liquid.

However, it was recognized by writers [8] that the key factor that controls the transfer in nucleate boiling is the stirring effect of bubbles, and that W_t can be neglected, compared with W_b while estimating their order. In view of the difficulty of obtaining directly the equation for heat transfer in nucleate boiling, it is conceivable that, with the rule of heat transfer in the case of free convection indicated by the following expression, an equation which uses Gr^* instead of Gr is applicable in the case of nucleate boiling.

$$Nu = K\,(Gr.Pr)^m \tag{10}$$

where

Nu = Nusselt number;
Pr = Prandtl number;
K and m = constants.

On the other hand, according to the writers' experiment [26], there exists the following relation between the coefficient of heat transfer α and δ as shown in Fig. 1:

$$\alpha^s \propto \delta^{-1}$$

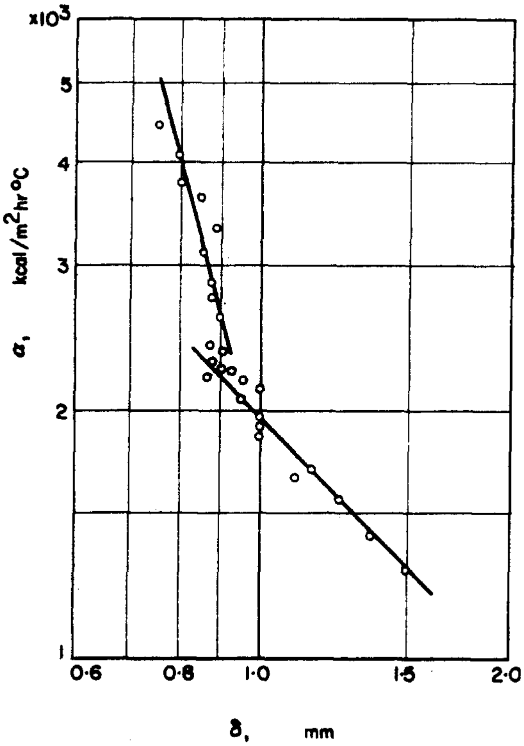

FIG. 1. Relation between the heat transfer coefficient, α, and thickness of the temperature boundary-layer, δ. [26].

Accordingly,

$$Nu^s \frac{\delta}{R} = B \tag{11}$$

is established, in which both s and B are constants. Therefore, neglecting Gr and substituting equations (9) and (11) into equation (10), the following formula will be obtained:

$$Nu = K^* \left(n\,Pr\,\frac{gH_e d_0^2}{\nu}\,\frac{\chi}{U_m}\right)^{m/(1-sm)} \tag{12}$$

where

$$K^* = (K)^{1/(1-sm)} \left(\frac{c_b}{c_t B}\right)^{m/(1-sm)} \tag{13}$$

Now, if the heat flux q is presumed to be carried away by bubbles, the number of vapour columns n is expressed by the following equation:

$$n = \frac{6}{r\gamma''}\,\frac{R^2 q}{d_u^3 f} \tag{14}$$

where

γ'' = specific weight of vapour;
r = latent heat of evaporation;
d_u = diameter of a bubble just leaving the free liquid surface (transformed into a sphere);
f = frequency of bubble formation.

However, the writers have verified by their experiment on water [17] that there exists the following relation between d_u and d_0:

$$\left(\frac{d_u}{d_0}\right)^3 = Mn^{-1/2} R \tag{15}$$

The numerical value of M in equation (15) is 900 m^{-1}, for boiling water under atmospheric pressure (see Appendix). Here, M is assumed to be a constant that has nothing to do with the physical properties of liquid, because it has been ascertained experimentally, as will be shown later, that M is a function only of the pressure. Substituting equation (15) into equation (14) and eliminating d_u, one can obtain the following expression:

$$n = \left(\frac{6}{r\gamma''}\,\frac{Rq}{Md_0^3 f}\right)^2 \tag{16}$$

Then U_m in the domain of heat transfer in nucleate boiling may be expressed experimentally by the following formula according to the study of Peebles and Garber [21].

$$U_m = b\left(\frac{\sigma g^2}{\gamma'}\right)^{1/4} \tag{17}$$

where

σ = surface tension of liquid;
γ' = specific weight of liquid;
b = constant which has nothing to do with the physical properties of liquid.

Now if distilled water is taken up as the standard when it boils under atmospheric pressure on a fresh and smooth plane and its value is expressed by the suffix *s*, the bubbles of any optional kind of liquid are considered to require ϕ times as much energy as those of distilled water when they leave the heating surface; in which case the formula will be as follows from Jakob's study [12]:

$$\phi = \left(\frac{d_0}{d_{0s}}\right)^3 \left(\frac{\gamma''}{\gamma_s''}\right) \left(\frac{r}{r_s}\right) \tag{18}$$

On the other hand, supposing that the frequency of bubble formation varies in accordance with the following expression:

$$f = f_s/\phi \tag{19}$$

the formula $\chi = 400$ m/hr = (constant) from the writers' experiment [26] will bring forth the following expression:

$$\chi = d_0 f = \frac{d_0}{d_{0s}}\,\frac{d_{0s} f_s}{\phi} = 400 \left(\frac{d_{0s}}{d_0}\right)^2 \frac{\gamma_s''}{\gamma''}\,\frac{r_s}{r}$$

Therefore $$\chi = P/(d_0^2 \gamma'' r) \tag{20}$$

where

$$P \equiv 400\, d_{0s}^2\, \gamma_s''\, r_s \tag{21}$$

According to the result of an experiment conducted by the writers [25] when air is injected into the liquid through one outlet, there is a distance from the heating surface at which the liquid stirring effect of the air bubble disappears when the air injecting outlet rises from the heating surface, and this distance varies according to the property of liquid. This distance is called "the effective stirring length of bubbles", and there also exists a similar length for nucleate boiling. The writers investigated the effect of the liquid level *H* on boiling heat transfer for some kinds of liquid, and found that the value of the coefficient of heat transfer for a certain liquid level remains constant, irrespective of the liquid level and begins to change below this level. Fig. 2 shows several examples. The liquid level where α begins to change is independent of heat flux for the specified liquid. Therefore, the writers have called the distance from the heating surface, which is equal to the liquid level where α begins to change, the effective stirring length of bubbles. The following relation holds between the effective stirring length of bubbles H_e and bubble Reynolds number $\bar{R}e_b$, as seen in Fig. 3:

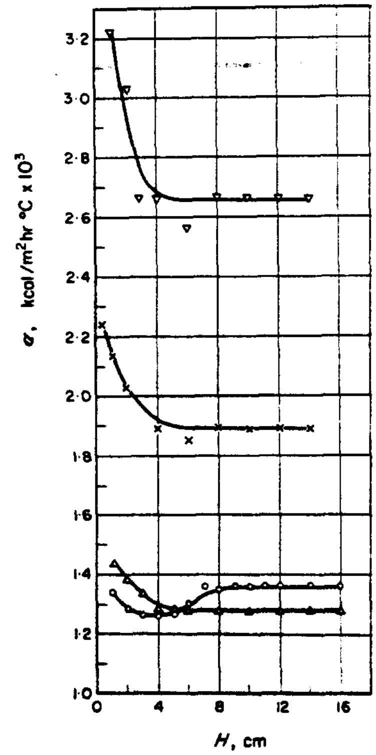

FIG. 2. Relation between the liquid level and the coefficient of boiling heat transfer.

	R (m)	$q \times 10^4$ (kcal/m²hr)	Liquid
▽	3·5	2·50	Distilled water
×	5·0	1·75	Distilled water
△	7·0	1·25	Distilled water
○	5·0	1·75	30% sugar water

$$\frac{H_e}{R} = T^* (\bar{R}e_b)^{-1} \equiv T^* \left(\frac{d_0 U_m}{\nu}\right)^{-1} \tag{22}$$

and d_0 is expressed by the following equation from Fritz's study [6]:

$$d_0 = a \sqrt{\left(\frac{\sigma}{\gamma'}\right)} \tag{23}$$

where T^* and a are proportional constants.

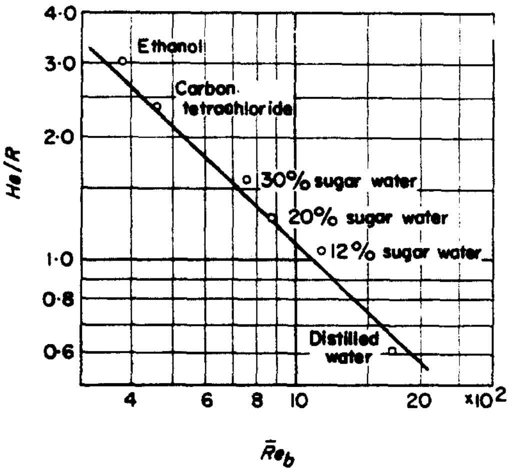

FIG. 3. Relation between the effective stirring length of bubbles, H_e, and bubble Reynolds number, $\bar{Re}_b$. [16].

According to the above formula, H_e decreases with increase in diameter of the bubble. This fact is hard to understand if only one bubble is considered. It should be kept in mind, however, that H_e is determined by the stirring effect of the total bubble which is the product of the average liquid stirring force for one vapour column and the number of vapour columns.

Finally, substituting equations (16), (17), (20), (22) and (23) into equation (12), the following equation is obtained:

$$Y = K^* \phi^* X^{2k} \tag{24}$$

where

$$Y \equiv \alpha R/\lambda; \tag{25}$$

$$\phi^* \equiv \frac{6^{2k}\, T^{*k}}{(ab^2)^k}; \tag{26}$$

$$X \equiv \left[\frac{1}{M^2 P}\, \frac{c_p \gamma'^2}{\lambda \sigma r \gamma''}\right]^{1/2} R^{3/2} q; \tag{27}$$

$$k = m/(1 - sm); \tag{28}$$

M = 900 m^{-1} (the value of a clean and smooth surface under the atmospheric pressure);

P = 400 $d_{0s}^2\, \gamma_s'' r_s$ = 1·699 kcal/hr;

c_p = specific heat of liquid at constant pressure;

λ = thermal conductivity of liquid.

As for R, the radius was taken to a horizontal circular plate, height to a vertical plate and diameter to a horizontal cylinder.

An attempt will be made to calculate the constant term by putting a suffix l in laminar flow and another suffix t in turbulent flow

$$K_l = 0{\cdot}56, \qquad K_t = 0{\cdot}13;$$

$$K_l^* = 0{\cdot}211, \qquad K_t^* = 0{\cdot}201;$$

$$m = \tfrac{1}{4},\ s = 1, \qquad m = \tfrac{1}{3},\ s = \tfrac{1}{4};$$

Therefore $k = \frac{1}{3}$, $\qquad k = \frac{4}{11}$.

And as $T^* = 1100$, $a = 1{\cdot}034$ and $b = 1{\cdot}18$, it will follow that

$$\phi_l^* = 30{\cdot}1, \qquad \phi_t^* = 41{\cdot}1;$$

Accordingly, the resulting formula will be

$$Y = 6{\cdot}35\, X^{2/3} \text{ (laminar)} \tag{29}$$

$$Y = 8{\cdot}26\, X^{8/11} \text{ (turbulent)} \tag{30}$$

and the experimental points will be contained almost between these two lines as seen in Fig. 5. As to the average for experimental points,

$$Y = 8{\cdot}0\, X^{2/3} \tag{31}$$

will be better. Since the numerical values determined by experiments at atmospheric pressure are used in deriving these formulae, equation (31) is the correlating equation of nucleate boiling heat transfer in the fresh and smooth surface under atmospheric pressure.

3. EFFECT OF PRESSURE

If the results of experiments which are either above or below the atmospheric pressure are correlated by the above-mentioned method, the experimental points will slip up in parallel with the straight lines of the formulae of equation (31) or (29) making the pressure a parameter (see, for example, Fig. 4). In order to unify these experimental points into one line, a definition will be made here for the pressure factor f_p as follows:

$$f_p = p/p_s \tag{32}$$

where

p = pressure in question;

p_s = atmospheric pressure.

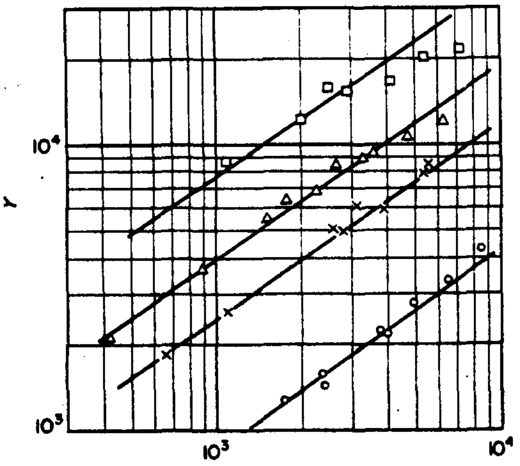

FIG. 4. Correlation of heat transfer in nucleate boiling for $X \sim Y$ at various pressures (data of Cichelli and Bonilla [3]).

Ethanol: ○, 1·03 atm; ×, 3·87 atm; △, 8·09 atm □, 18·63 atm.

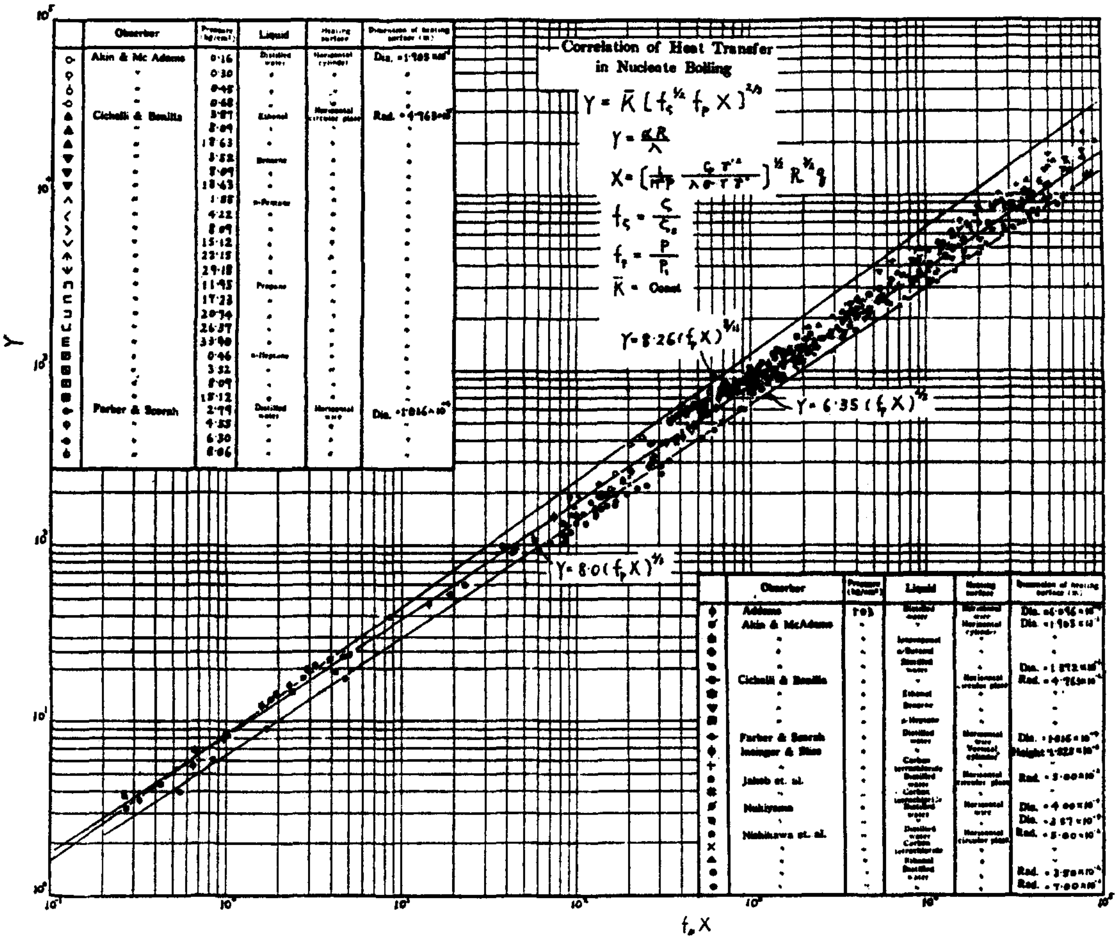

FIG. 5. Correlation of heat transfer in nucleate boiling.

P

The formula (31) may be rewritten as follows, using this pressure factor:

$$Y = 8{\cdot}0\,(f_p X)^{2/3} \qquad (33)$$

If the data of the writers [26], Jakob [11], Cichelli and Bonilla [3], Addoms [1], Insinger and Bliss [10], Akin and McAdams [2], Nukiyama [20] and Farber and Scorah [5] are plotted out by adopting Y as ordinate and $f_p X$ as abscissa, Fig. 5 is obtained and the experimental points will be unified with 20 per cent on the whole.

Since the variation of the physical properties of the liquid with pressure is taken into account in the terms X and Y of equation (33), it is considered that the pressure factor f_p is the correction factor of M in equation (15). Eliminating d_u from equations (14) and (15), an expression will be formed as follows:

$$M = \frac{1}{\sqrt{(n)}}\left(\frac{6}{r\gamma''}\,\frac{Rq}{d_0^3 f}\right) \qquad (34)$$

On the other hand, the following expression must hold, assuming that f_p is the correction factor of M only,

$$M p = M_s p_s \qquad (35)$$

Here, the suffix s refers to the values at atmospheric pressure. In other words, the factor concerning the rate of growth of bubble M is inversely proportional to pressure p. As the writers [18] measured not only α and q but also n, d_0 and f in the experiment of nucleate boiling of water under reduced pressures, one can calculate M from equation (34). Plotting the relation between p and M determined by equation (34), Fig. 6 is obtained and it is ascertained that the relation (35) holds. The right-hand side of equation (34) contains the physical properties of liquid and the constants which must be determined by a boiling experiment and vary according to the kind of liquid. Therefore, it can be said that the pressure factor f_p is the correction factor which modifies only the factor concerning the rate of growth of the bubble with pressure, and the assumption in the preceding paragraph in which M is only a function of the pressure, independent of properties of liquid, could be indirectly proved.

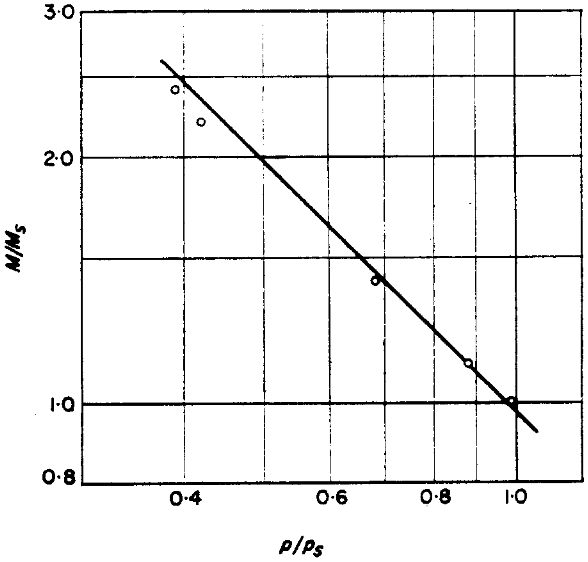

FIG. 6. Relation between rate of bubble growth, M, and pressure, p. [18].

4. EFFECT OF CONDITION OF HEATING SURFACE

Since formula (33) is based on a fresh and smooth surface as mentioned before, consideration should be given to various other kinds of surface conditions, e.g. surfaces that are contaminated, roughened or air-adsorbed, in regard to the foaming capacity of the surface as has been pointed out. The writers previously carried out an experiment [9] on nucleate boiling by using water containing salt and obtained a result as follows. When the heat flux q increases $q_1 \rightarrow q_2 \rightarrow q_3$ and decreases as $q_3 \rightarrow q_2 \rightarrow q_1$ and if these cycles are repeated, the degree of contamination on the surface is considered to be definite during one cycle. If the result of the experiment is expressed by the curve $\alpha \sim \Delta\theta$ ($\Delta\theta$ = the temperature difference between heating surface and liquid), it will resemble Fig. 7, i.e. it will produce $\alpha \propto \Delta\theta^2$ which will go on changing its position every time the cycle is repeated, maintaining the preceding relation. Accordingly, ζ which is defined in equation (36) expresses the foaming capacity of the specified liquid:

$$\zeta \equiv \alpha R/\Delta\theta^2 = (\alpha R)^3/(qR)^2 \qquad (36)$$

When the physical properties are different, it is necessary to have a separate term which indicates nothing but the foaming capacity, for ζ is

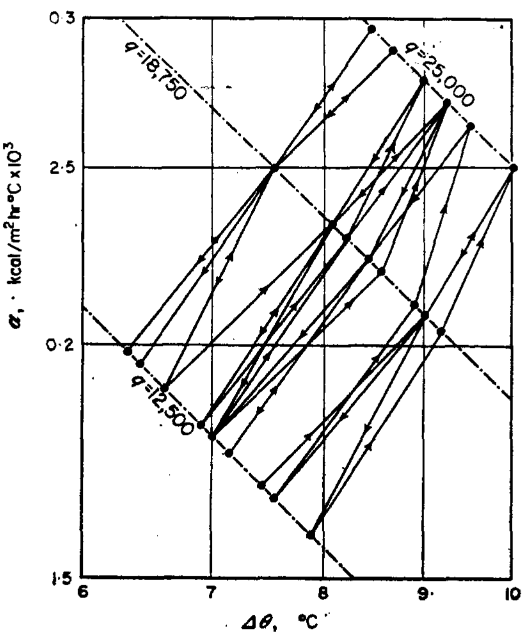

FIG. 7. Change of the relation $\alpha \sim \Delta\theta$ due to the repetition of cyclic heating [9].

included in the effect of physical properties. Therefore, if f_ζ (see equation (37)) is defined as foamability, it may be looked upon as the factor that indicates only the foaming capacity in a form from which the effect of physical property has been eliminated:

$$f_\zeta = \zeta/\zeta_s \qquad (37)$$

where ζ_s is the coefficient of foaming capacity of pure liquid on the fresh and smooth surface while ζ represents the coefficient of foaming capacity of a combination of the surface and the liquid in question. The formula of heat transfer in nucleate boiling, in which the condition of the heating surface is considered, may be obtained by rewriting the formula (33) as follows using f_ζ:

$$Y = 8{\cdot}0\,(f_\zeta^{1/2} f_p X)^{2/3} \qquad (38)$$

That is to say, formula (38) is an expression that includes all the cases of heat transfer in nucleate boiling. The foamability is something that resembles to emissivity in heat radiation, which is decided by a combination of the condition of the heating surface and of the liquid.

Generally, when the heating surface is contaminated, foaming capacity either increases or decreases. But, it is sufficient to regard $f_\zeta = 1$ in an ordinary contaminated or roughened surface, but not in a very contaminated or artificially grooved surface. Though it is necessary to determine f_ζ experimentally in order to know the accurate value of the coefficient of heat transfer, in many cases it is sufficient to take $f_\zeta = 1$ in design practice.

5. APPLICATION TO FORCED CONVECTION BOILING HEAT TRANSFER

5.1. *Saturated boiling*

In a case where the saturated liquid flows in a tube, the value of the coefficient of heat transfer becomes somewhat larger than that of pool boiling heat transfer, for which Rohsenow

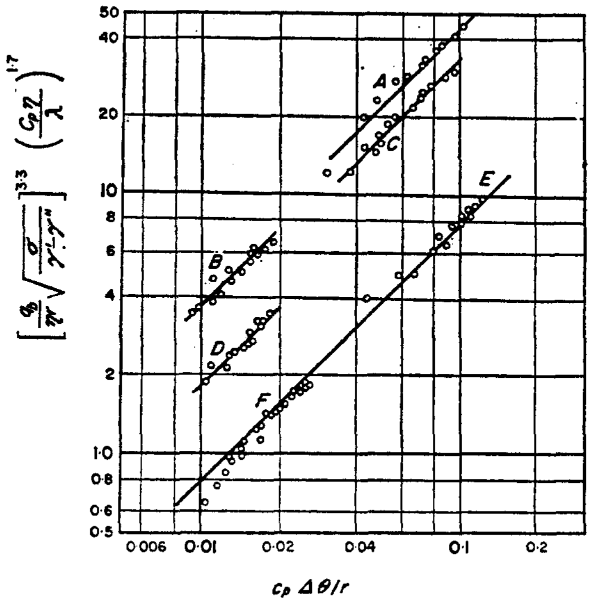

FIG. 8. Correlation of boiling heat transfer inside tubes by Rohsenow's method [22].

		C_{sf}
A	*iso*Propyl alcohol	0·00225
B	50% potassium carbonate	0·00275
C	n-Butyl alcohol	0·00305
D	35% potassium cabonate	0·0054
E	Carbon tetrachloride	0·013
F	Water	0·013

[24] has suggested a correlating method such as follows, that is, the total heat flux q at forced-convection boiling consists of boiling heat flux q_b and convective heat flux q_c:

$$q = q_b + q_c \tag{39}$$

q_c will be calculated by the conventional Dittus–Boelter equation, modified by using a coefficient of 0·019 instead of 0·023.

$$q_c \quad = \alpha_c \Delta\theta \tag{40}$$

$$Nu_c = 0{\cdot}019\ Re^{0{\cdot}8}\ Pr^{1/3} \tag{41}$$

where

α_c = coefficient of convective heat transfer;
Nu_c = Nusselt number $\equiv \alpha_c D/\lambda$;
D = diameter of tube;
Re = Reynolds number $\equiv vD/\nu$;
v = velocity of fluid;
Pr = Prandtl number.

As for q_b, Rohsenow uses a correlating equation of his own, i.e.

$$\frac{c_p \Delta\theta}{r} = C_{sf} \left\{ \frac{q_b}{\eta r} \sqrt{\left(\frac{\sigma}{\gamma' - \gamma''} \right)} \right\}^{0{\cdot}33} (Pr)^{1{\cdot}7} \tag{42}$$

where

η = viscosity of liquid;
C_{sf} = surface coefficient determined by the combination of liquid and heating surface (function of contact angle).

The experimental values of each liquid or each experimental condition can be expressed by a group of paralleles if correlated by Rohsenow's formula, as shown in Fig. 8, but it causes a serious difference to the value of surface coefficient C_{sf}. Accordingly, the writers followed up q_b by applying their correlating equation (31) to the results of an experiment conducted by Piret and Isbin [22]. The latter took several liquids of different physical properties, and caused them to flow through a vertical copper tube, 25·4 mm in diameter and 1·46 m in length, boiling under atmospheric pressure. We obtained an excellently unified result which is indicated in Fig. 9. The reason why data of 30 per cent and 50 per cent potassium carbonate are omitted in Fig. 9 is that these data are considered for the contaminated surface. As a result, it may be said that Rohsenow's conception on the forced-convection boiling heat transfer is approximately correct

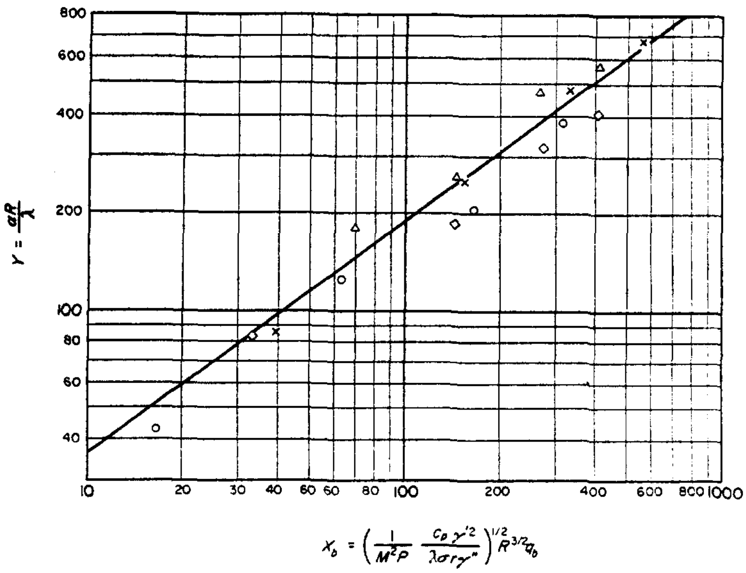

FIG. 9. Correlation of boiling heat transfer inside tubes by writers' method. ○, water; ×, *iso*propyl alcohol; △, *n*-butanol; ◇, carbon tetrachloride.

where he believes the total flux in the forced-convection boiling to be the sum of convective heat flux and boiling heat flux; however, in Rohsenow's coefficient of surface factors are included conditions other than that of interface between solid and liquid.

If the line of thought shown in formula (39) is applied, the liquid boiling inside the tubes will be satisfactorily correlated by using the writers' correlating equation, but it can be applied only to cases in which vapour and liquid are quite evenly mixed. In those cases where vapour is formed into a piston by the excess of vapour, such a way of thinking is invalid. It will be necessary to make further studies on the flowing aspect of vapour–liquid mixtures in these instances.

5.2. *Surface boiling*

Rohsenow holds that the correlating equation he has proposed for saturated boiling is also applicable to surface boiling, but the writers are not satisfied with it for the following reasons:

(1) In the process of deriving this empirical formula, it is assumed $n \propto q_b$ but according to experiments carried out by the writers, it turned out that $n \propto q_b^2$.

(2) Though Rohsenow and Clark state that the contamination of the heating surface causes a change in the exponent of Pr, the writers believe that the effect of the condition of the heating surface ought to be included in the surface coefficient C_{sf} but not in the exponent of Pr.

(3) According to the former, the value of C_{sf} is different for surface boiling and for saturated boiling respectively, even if the combination of the liquid and the heating surface is the same, but the writers presume that C_{sf} ought to assume the same value for surface boiling as for saturated boiling.

The writers tentatively co-ordinated the experimental results on heat transfer in surface boiling based on the correlating equation which they had proposed for pool boiling, though it seems to be a questionable treating of the mechanism of surface boiling to apply a correlating equation which uses the concept of the effective stirring length of bubbles.

First, assuming equations (39), (40) and (41) valid, q_b was evaluated. Next, $q - \Delta\theta$ diagrams are often used in treating the data of surface boiling. When such a correlation is used, several parallel lines are formed having the subcooling degree of the liquid as a changing parameter. If $q - \Delta\theta_s$ diagrams are used, these parallel lines can be represented by a single curve, where $\Delta\theta_s$ is surface temperature θ_p minus saturation temperature θ_s. From these results as well as from the experimental results on the formation and collapse of bubbles in surface boiling, the driving force in the surface boiling heat transfer is considered as $\Delta\theta_s$. The writers used heat flux as a measure of the driving force of heat transfer in place of temperature difference. Thus the writers tentatively used $q_{bs} \equiv q_b(\Delta\theta_s/\Delta\theta)$ as the heat flux in X of equation (27). A satisfactory correlation could not be obtained by this treatment alone, though this is to be expected. Accordingly, the writers accounted for the correction of the effective stirring length of bubbles.

In the writers' correlating equation corresponding to the saturated boiling, the effect of the effective stirring length of bubbles is included in the form of $(T^*)^{1/3}$ in the ϕ^* term of equation (24). T^* is the constant connecting the effective stirring length of bubbles to the bubble Reynolds number, and it satisfies equation (22). The writers considered that the value of the effective stirring length of bubbles in surface boiling differs from that in saturated boiling, even in the same liquid, and modified equation (22) as follows:

$$\frac{H_{es}}{R} = T T^* (\bar{R}e_b)^{-1} \tag{43}$$

Therefore

$$T = H_{es}/H_e \tag{44}$$

where H_{es} is the effective stirring length of bubbles in surface boiling. Though ϕ^* in equation (24) should be modified by this correction factor T, the writers corrected X for the sake of convenience.

Now, according to the experiments of Gunther [7] and Rohsenow and Clark [23], the maximum diameter of generated bubbles is 3 mm at most and about 1 mm in the mean diameter. On the other hand, since the effective stirring length of bubbles determined on saturated water under

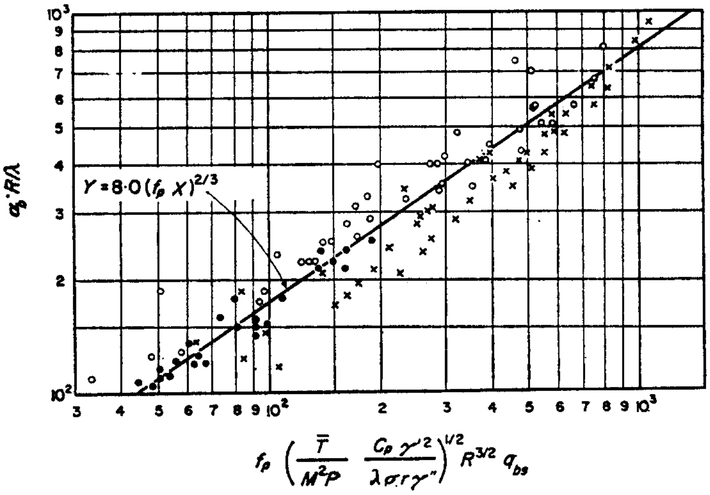

FIG. 10. Correlation of surface boiling heat transfer by the writers' method.

	Authors	p (atm)	v (m/sec)	$\Delta \theta_s$ (°C)	D and R (m)
○	Kreith–Summerfield	1–15	1·5–4	50–150	$1{\cdot}49 \times 10^{-2}$ (D)
×	Rohsenow–Clark	140	3–9	60–150	$4{\cdot}59 \times 10^{-3}$ (D)
●	Nishikawa–Urakawa	1	0	0–7	0·05 (R)

atmospheric pressure by the writers [16] is 30 mm. A satisfactory result has been obtained for saturated boiling inside tubes, in which case the effective stirring length of bubbles determined in the horizontal heating surface is used. Presuming the effective stirring length of bubbles in surface boiling is of the same order of magnitude as the diameter of bubbles, it may be reasonable to assume that $\bar{T} = d_0/H_e = 3/30 = 1/10$ for water.

The experimental data of Kreith and Summerfield [13] and of Rohsenow and Clark [4] were treated in this way, and the results are shown in Fig. 10. As seen in the figure, the experimental points are not only favourably arranged in a straight line, but are strongly suggestive of the fact that both surface boiling and saturated boiling can be represented by a single straight line. The same experimental data were treated by the formula of Rohsenow and the results are shown in Fig. 11.

The value of $\bar{T}$ may depend upon the degree of subcooling, the flowing condition and the heat flux. The writers [19] carried out the experiment on free convection surface boiling in order to clarify the effect of the degree of subcooling. They found that the relation shown in Fig. 12 holds between the rising height of bubble h, i.e. the height to which bubbles could rise without disappearance, and the degree of subcooling $(\theta_s - \theta_L)$. Since Fig. 12 was obtained from an experiment for a narrow range of heat flux, the data should be correlated by taking the heat flux as the changing parameter in the case of a large range of heat flux. In surface boiling, a bubble that has been generated on the heating surface condenses on its way up. The stirring effect of bubbles therefore gets smaller and smaller for two reasons, i.e. that the bubble goes up from the heating surface and that the size of bubble becomes smaller. Consequently, the writers assume that the effective stirring length

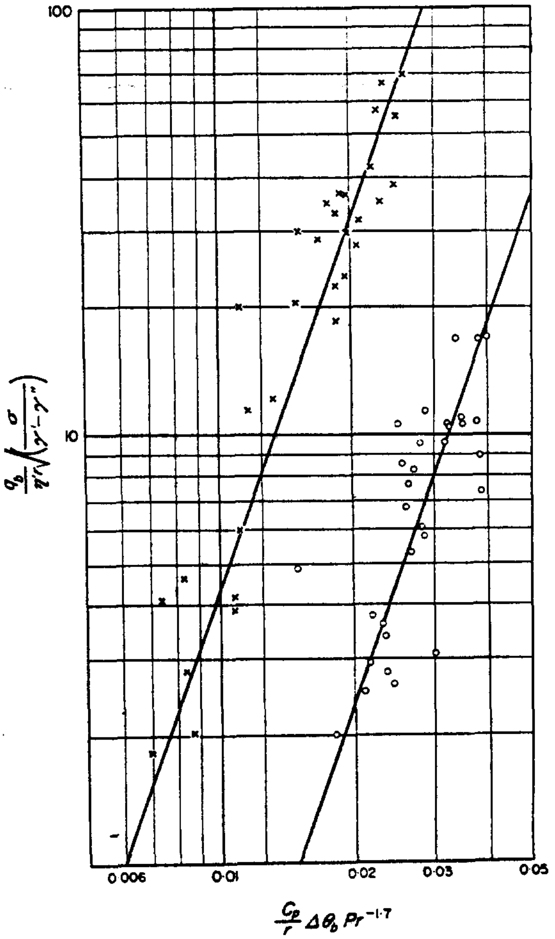

FIG. 11. Correlation of surface boiling heat transfer by Rohsenow's method.

○, Kreith-Summerfield; ×, Rohsenow-Clark.

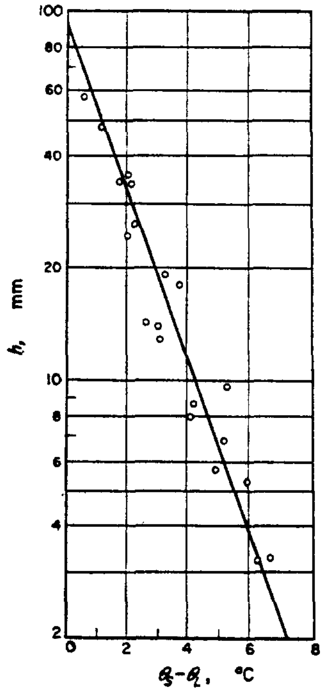

FIG. 12. Relation between rising height of bubble, h, and subcooling degree of liquid, $\theta_s - \theta_L$. (q = 15,000–26,000 kcal/m² hr).

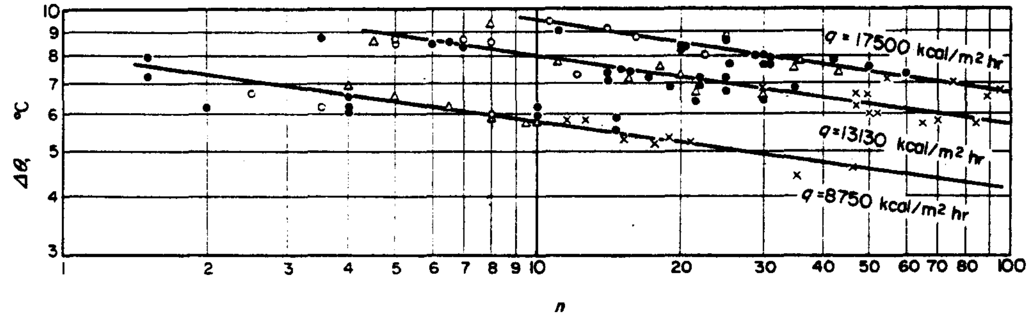

FIG. 13. Relation among the number of vapour columns, the temperature difference between the heating surface and liquid and the heat flux.

○, distilled water; ●, saponin solution; △, sodium oleate solution (3 p.p.m.); ×, sodium oleate solution (15 p.p.m.).

of bubbles is equal to the height of the circular cylinder that has the same volume of cone with one vapour column replaced. The result based on the above-mentioned assumption is plotted in Fig. 10.

The above-mentioned correlating method is only an example of the application of the writers' correlating equation. The behaviour of bubble generation in surface boiling differs from the case of saturated boiling. Therefore it is necessary to clarify the mechanism of heat transfer in surface boiling in order to obtain the correct correlation of surface boiling heat transfer.

6. CONCLUSION

The writers have logically derived a correlating equation of heat transfer in nucleate boiling for various kinds of liquids whose physical properties are different and indicated that this correlating equation is applicable to forced-convection saturated boiling or surface boiling. The writers' correlating equation has been obtained theoretically by analysing the elementary processes of the phenomena and is not based on a dimensional analysis. It is, therefore, considered that a good correlation between the proposed equation and the data of heat transfer shows the adequacy of the results of bubble dynamics obtained by the writers, and it is necessary to analyse the elementary processes of boiling phenomena and elucidate the relations underlying them in order to reach a final solution to the problem.

APPENDIX

The value of M in equation (15) for boiling water under atmospheric pressure is determined as shown below. Equation (14) can be rewritten as follows:

$$qR^2 = \left(\frac{r\gamma''}{6}\right) n d_u^3 f = \left(\frac{r\gamma''}{6}\right) n d_0^3 f \left(\frac{d_u}{d_0}\right)^3 \qquad \text{(A1)}$$

As seen from Fig. 13, the following relation holds for boiling water under atmospheric pressure:

$$\Delta\theta = c_q\, n^{-1/6} (qR)^{2/3} \qquad \text{(A2)}$$

where $\Delta\theta = \theta_p - \theta_L$ and c_q is the constant which varies according to the type of liquid.

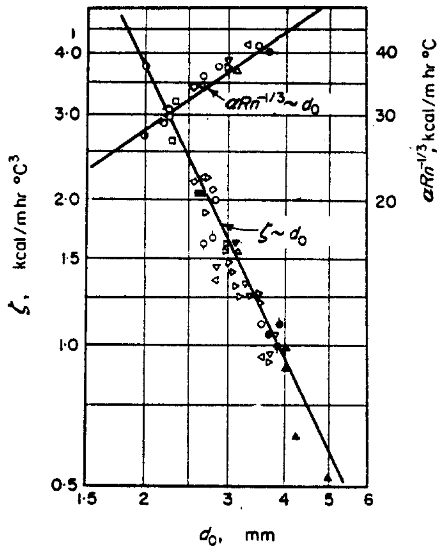

FIG. 14. Relation between bubble diameter just leaving the heating surface and foamability.

○, roughened surface with concentric grooves (average depth of groove, 0·1701 mm);
△, roughened surface with concentric grooves (average depth of groove, 0·0846 mm);
×, roughened surface with concentric grooves (average depth of groove, 0·0425 mm);
◎, roughened surface with concentric grooves (average depth of groove, 0·0090 mm);
◇, roughened surface with concentric grooves (average depth of groove, 0·0750 mm);
□, roughened surface with concentric grooves (average depth of groove, 0·1299 mm);
▽, roughened surface with concentric grooves (average depth of groove, 0·0192 mm);
▷, roughened surface with concentric grooves (average depth of groove, 0·0578 mm);
■, roughened surface with concentric grooves (average depth of groove, 0·0048 mm);
▼, roughened surface with crosswise grooves (average depth of groove, 0·0375 mm);
o, Nekal *X* water solution (Jakob);
▲, sugar water (Urakawa);
●, smooth surface $R = 5$ cm;
●, smooth surface $R = 7$ cm;
●, smooth surface $R = 3{\cdot}5$ cm;
o-, water solution of saponin;
♀, sodium oelate solution (3 p.p.m);
-o, sodium oleate solution (15 p.p.m.).

Equation (A2) can be rewritten as follows:

$$\alpha\, R = \left(\frac{1}{c_q}\right) n^{1/6} (qR)^{1/3} \qquad \text{(A3)}$$

On the other hand, equation (12) in laminar flow will be rewritten as follows by combining the physical constants and the variables which are considered not to change for a specified liquid:

$$\alpha R = c_K n^{1/3} (d_0^3 f)^{1/3} = c_K \chi^{1/3} n^{-1/3} d_0^{2/3} \quad \text{(A4)}$$

where

$$c_K \equiv \lambda K^* \left(Pr \frac{g H_e}{\nu^2} \frac{1}{U_m} \right)^{1/3}$$

Eliminating α from equations (A3) and (A4), the following expression can be obtained:

$$n = \left(\frac{1}{c_q c_K} \right)^6 (d_0^3 f)^2 (qR^2) \quad \text{(A5)}$$

Eliminating q from equations (A1) and (A2), the following equation can be obtained finally:

$$\left(\frac{d_u}{d_0} \right)^3 = M n^{-1/2} R \quad \text{(A6)}$$

where

$$M = \left(\frac{6}{r \gamma''} \right) (c_K c_q)^3 n^{-1/2} R$$

Since $c_q = 0{\cdot}149$ mhr$^{2/3}$°C/(kcal)$^{2/3}$, $r = 538{\cdot}8$, kcal/kg, $\gamma'' = 0{\cdot}598$, kg/m^3, $\chi = 400$ m/hr and $c_K \chi^{1/3} = 1800$ kcal/m$^{5/3}$hr°C (from Fig. 14) for the boiling water under the atmospheric pressure, the numerical value of M in this case is $M \doteqdot 900$ m^{-1}.

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